1 V SGE PROPERTIES

Constellation = Sagitta, The Arrow (best visible on summer evenings)
Coordinates = 20:20:14.69, +21:06:10.4, right ascension and declination, J2000
$D = 7760^{+750}_{-460}$ light-years, distance
$\langle V_{2019} \rangle = 11.0$ mag, average visual brightness in the year 2019
$\langle B - V \rangle = 0.05$ mag, average color
$E(B - V) = 0.20 \pm 0.05$ mag, excess $B - V$ color
M_V = -2.2 mag, average absolute magnitude
$P = 0.514212053$ days = 12.34 hours, orbital period
$\dot{P} = -2.44 \times 10^{-10}$ days/cycle, rate of change of the orbital period
$|\ddot{P}| < 5 \times 10^{-15}$ days/cycle^2, acceleration of orbital period
$M_{WD} = 0.85 \pm 0.05$ M , mass of the white dwarf
$M_{comp} = 3.3 \pm 0.2$ M , mass of the normal companion star
$q = 3.9 \pm 0.3$, mass ratio, M_{comp}/M_{WD}
$T_{comp} = 12{,}000^{+8000}_{-2000}$ K, surface temperature of companion star
$\dot{M}_{2000} = (1\text{-}3) \times 10^{-5}$ M $/yr$, accretion rate in year 2000
$\dot{M}_{2000}/\dot{M}_{1900} = 2.2 \pm 0.2$, factor increase of accretion between years 1900 and 2000
$\dot{M}_{wind,2000} = (0.55\text{-}2.3) \times 10^{-5}$ M $/yr$, stellar wind rate in year 2000
$V_{wind} = 1500^{+2500}_{-500}$ km/s, surface temperature of companion star

2 THREE PROOFS THAT V SGE IS IN-SPIRALING VERY FAST

There are three independent and strong proofs that the orbit of V Sge has the orbital period and radius decreasing rapidly, making the orbital path spiral inwards:

2.1 Mass Ratio >> 1

The first proof starts with the confidently measured masses of both the companion star and the white dwarf. (V Sge is a double-line spectroscopic binary and an eclipsing binary, so the masses are confidently known.) The mass ratio is q=3.9±0.3. It has long been known that systems in contact with q>1 are necessarily unstable to runaway mass transfer, due to an instability that causes the in-spiral.

The physical mechanism for this runaway instability is easy to see. At some given time, when a relatively massive companion transfers mass, its Roche lobe must necessarily get smaller, increasing the mass transfer rate, which then makes the Roche lobe shrink even faster, which makes the mass transfer rate increase even faster, which makes the companion's Roche lobe shrink even faster, and so on. This instability has been long known, but no one has ever seen such a star system, other than V Sge.

V Sge is the *only* known Cataclysmic Variable (CV) that has the q>1 case. All other CVs have q<1, and even the few so-called 'V Sge stars' all have q<0.5. So V Sge is the unique case where this runaway instability is known to occur. With all other CVs being in a completely different regime of mass transfer, all common experience and theory from the CV community is completely inapplicable to V Sge.

2.2 Period is Observed to Be Decreasing Fast

V Sge is an eclipsing binary, where the time of the minimum eclipse is a close and accurate measure of the time of conjunction in the orbit. By timing between two minima, the orbital period, P, can be measured. If the eclipses are widely separated (and the cycle count can be kept, which is straight forward), the P can be measured with remarkably high accuracy. By collecting eclipse times over many decades, we can watch the orbital period evolve. V Sge has had eclipse times measured and put into the literature photoelectrically from 1962 to 1994, and with CCDs up until 1998. These eclipse times in the literature have been supplemented with (a) eclipse times from archival photographic plates, going back to 1909, (b) amateur CCD light curves from 2000 to 2018, and (c) with the awesome TESS light curves from July and August 2019. These 183 eclipse times from 1909–2019 show a highly significant and blatant and steady change in the orbital period. This change can be labeled $\dot{P}$. The measured quantity is $\dot{P} = -2.44 \times 10^{-10}$ days/cycle. The negative sign means that the period is *decreasing* over time. This is measured to 2.0% accuracy.

So we have direct observations over 111 years that the orbital period is decreasing. Indeed, with this being the most negative measured $\dot{P}$ for any CV, we see that the orbit is decreasing *fast*. That is, V Sge is in-spiraling *fast*.

2.3 System is Brightening With a Doubling Time Scale of 89 Years

V Sge was discovered in the year 1904, and there have been visual magnitude estimates continuously since then. Further, V Sge has been recorded on now-archived old sky photographs from 1890 on, providing another record of the brightness variations. Care must be taken so that all the individual magnitudes are accurately placed onto a consistent and modern magnitude system, and this is straight forward to do with old comparison sequences and modern magnitudes. The result is a fully-modern V-band light curve with 68,016 magnitudes from 1904–2019, plus a light curve of 1531 B magnitudes from 1890–2017.

Both of these light curves independently show that V Sge is systematically brightening from 1890 to 2019. This systematic brightening has chaotic variability superposed, and the highly significant brightening trend is not perfectly linear. The star has brightened by a factor of 2.2±0.2 from 1900 to 2000. The average rate of brightening is -0.84 magnitudes per century. in the V band and -0.92 magnitudes per century in the B band. A linear rise in magnitudes over time is the same thing as an exponential increase in flux. This can be expressed as a 'doubling time scale'. For V Sge, the average doubling time scale is 89±11 years. It turns out that the measurement uncertainties in this doubling time scale are the dominant source of uncertainty for predicting the future year of the merger.

So the brightness of V Sge is increasing with a doubling time of 89 years or so. The brightness of V Sge is entirely dominated by the accretion luminosity. So this means that the accretion rate is doubling with a time scale of 89 years. The only way to get the accretion rate to rise so fast is for the Roche lobe of the companion star to be shrinking fast, and this means that the orbital separation must be shrinking fast. That is, the orbit of V Sge is in-spiraling fast.

3 HOW CAN THE MERGER HAPPEN IN ONLY SIX DECADES OR SO?

So V Sge is in-spiraling fast, but this does not say *when* the inevitable merger will be. This section will provide the justification for why the final merger can and will happen around the year 2083 AD or so.

3.1 The Merger is Not in Millions of Years

For ordinary CVs, the standard approximate measure of the time till merger is simply $|P/\dot{P}|$. For V Sge, this time scale is 2.1×10^9 cycles or close to 3 million years. So this simple time scale has the merger millions of years from now.

Such a time scale is reasonable for all CVs with $q < 1$, but is completely inapplicable for the unique case of q=3.9. The reason is that the period and Roche lobe size will indeed initially decrease linearly with this time scale, but the *accretion rate* will increase exponentially with this linear decrease in the Roche lobe size. What is going on is that the Roche lobe is eating into the companion star while the gas density in the companion's atmosphere is rising exponentially with depth. This is just the usual Law of Atmospheres required for hydrostatic equilibrium. We only care about a few thousand kilometers of the star's atmosphere, for the few decades bebfore the end and the Law of Atmospheres applies well for such small ranges over which the temperature and gravity change little. The exponential atmospheric scale height of the companion star is close to 750 km. If the atmosphere were to remain static as the Roche lobe eats into it, larger and larger amounts of mass would fall off the companion, increasing exponentially with the depth of penetration. Still, there are complications due to the non-static upper atmosphere being lifted off by the Roche lobe overflow, which result in the atmosphere rising in response to the removal of its top layers. As a result, the mass transfer rate rises exponentially with the amount of overflow, which in turn increases proportional to the speed of penetration of the Roche lobe and the speed of expansion of the atmosphere. We can therefore summarize the above discussion by stating that the mass transfer rate $MT(t)$ varies according to:

$$MT(t) = MT(0)\exp\left[(R2(t) - R_{\rm Roche}(t))/H\right], \tag{1}$$

where $MT(0)$ is the transfer rate at some arbitrary reference time, $R2$ is the radius of the photosphere of the companion, and $R_{\rm Roche}$ is the Roche radius. Therefore, $R2 - R_{\rm Roche}$ measures the time-dependent depth of penetration of the Roche lobe into the exponential atmosphere of the companion which has the scale height $H \approx 750$ km.

So let us see the time progression of the accretion rate as the Roche lobe size decreases, initially nearly linearly with time, and even faster later. Over some time interval during which the depth $R2 - R_{\rm Roche}$ increases by 750 km, some specific amount of mass will fall off the companion. Over the next nearly equal-time interval, the Roche lobe is eating into gas layers that are e=2.7 times denser, so this interval will have the accretion increase by a factor of 2.7X. Over the next equal-time interval, the Roche lobe eats into yet denser layers of gas in the star's atmosphere and will be 2.7X that of the previous interval and 7.4X that of the first interval. As the Roche lobe decreases, and the star expands, the accretion rate rises exponentially. This rise of the accretion rate is what drives the runaway instability. So the runaway instability occurs on a time scale greatly shorter than $|P/\dot{P}|$. Analytically, from standard equations as applied to this unique system, we expect the characteristic time scale to be $\sim H/R_{Roche}$ times $|P/\dot{P}|$. Let us label this time scale as τ. As $R_{\rm Roche}$ is greatly larger than H, τ will be greatly faster than three million years. From this, the accretion rate will vary with time as some rapidly varying function of t/τ. See the next section for a more accurate discussion of this time scale and a simple derivation of the functional form of $MT(t)$. With the accretion rate rising on a fast time scale, the merger will be driven on a time scale of greatly faster than a million years.

3.2 The Merger Will Be In the year 2083 ± 16

The future of V Sge is straight forward to predict with good accuracy by the simple use of standard equations that model all the various physical mechanisms involved. The idea is to start with the well-known current system properties, use these properties to calculate how each of the properties will change in the next time interval, add these changes to get the system properties for a year or a day from now, and keep repeating this for time intervals marching off into the future. Initially, it imay be adequate to have one-year intervals, but as the end approaches, shorter and shorter intervals are needed. The changes in angular momentum, semi-major axis, and Roche lobe size are governed by the mass transfer rates, and the changes in the mass transfer rates are governed by the rate of decrease for the Roche lobe radius. So, by propagating the system into the future, year-by-year and day-by-day, the future properties of V Sge can be predicted. This has been done by setting up a system of first-order differential equations governing the binary separation, the masses of the binary component stars, and the radius of the companion star. With these equations we can advance in time from some specified initial state and thus predict the evolution of the system.

The key result is that the accretion rate rises at first nearly exponentially with time, but much faster later as the critical (merger) time is approached. The basic set of calculations, using the best measures for all the system properties and and matching all observations, has been performed independently in different codes by different workers. The baseline result is that V Sge will merge close to the year 2083. A formal error bar can be determined by varying all input parameters by their known error bars. This gives an error bar of ±16 years for the 2083 date. Almost all of this arises solely from the uncertainty in the 2.2±0.2 factor for the accretion rate in 2000 versus 1900. Importantly, the merge year changes little for all other changes in input. For example, if the white dwarf mass is increased from 0.85 to 0.90 M (all within the uncertainties), then to get the observed factor of accretion rate increase to be 2.2, the accretion at the fiducial year must decrease (all within the allowed range from observations), and this makes for the best fit with the different white dwarf mass to exactly equal 2083. That is, the merge year of 2083 is robust against changes in the assumed white dwarf mass. Similarly, the merge year is robust to changes for all other input parameters (other than the 2.2 factor). So the predicted merge year is 2083±16, and this is robust for changes on all system parameters.

There is a simple albeit approximate way to show that these results are robust and that the above mentioned ratio of mass transfer rates essentially determines the time of the merger. We begin by differentiating equation (1) with respect to time:

$$\dot{MT}(t) = MT(0)\exp\left[(R2 - R_{\rm Roche})/H\right](\dot{R}2 - \dot{R}_{\rm Roche})/H = MT(t)(\dot{R}2 - \dot{R}_{\rm Roche})/H. \tag{2}$$

The dots indicate time derivatives, and in obtaining the above result we have assumed that the scale height H remains constant. Following standard practice in approximate analytic treatments of mass transfer in Cataclysmic Variables and Low-Mass X-ray Binaries, we write the change of $R2$ and $R_{\rm Roche}$, in response to the mass transfer as follows:

$$\frac{\dot{R}2}{R2} = -\zeta_2 \frac{MT}{M_2} \qquad \text{and} \qquad \frac{\dot{R}_{\rm Roche}}{R_{\rm Roche}} = -\zeta_{\rm Roche} \frac{MT}{M_2} \tag{3}$$

where the ζs are the logarithmic derivatives of the respective radii with respect to the mass of the donor, noting that MT is the *magnitude* of the mass transfer, and therefore it is a mass *loss* from the companion, $\dot{M}_2 = -MT$. While it is possible to assign values to ζ_2 and $\zeta_{\rm Roche}$ based on physical considerations, their exact values are not important for the following argument, only the sign of $(\zeta_2 - \zeta_{\rm Roche})$. With these definitions, equation (2) becomes

$$\dot{MT}(t) = -(\zeta_2 - \zeta_{\rm Roche})\frac{R_2}{H}\frac{MT^2}{M_2} \tag{4}$$

During the evolution of the system, the ratio R_2/H and the mass of the companion M_2 do not vary very much until the very last days before the merger, so we are justified in treating them as constants in time and integrating the above equation (4). Without any loss of generality we can write $MT = MT(0)y(t)$, where y is a dimensionless function of time, and choose $MT(0)$ to be the mass transfer in 1900. Then we have

$$\frac{\dot{y}}{y^2} = -\frac{1}{\tau} \qquad \text{where} \qquad \tau = \frac{H}{R_2}\frac{M_2}{MT(1900)}(\zeta_{\rm Roche} - \zeta_2)^{-1} \tag{5}$$

where τ is a characteristic time for the evolution of MT,. Under the assumption made thus far τ is a constant and the integral is immediate yielding a simple result:

$$y(t) = 1/(1 - (t - 1900)/\tau) \tag{6}$$

What is the value of τ that makes $y(2000) = 2.2$? The answer is also simple and surprising given the assumptions of the model, $\tau = 183.3$ yr, predicting that the mass transfer will diverge as the system merges at $t = 2083.3$! Notably this depends only on the much discussed ratio $MT(2000)/MT(1900)$. If we allow it to vary in the range 2 to 2.4, we obtain $\tau = 200$ yr and 171.4 yr respectively implying merger times between 2071 and 2100. Essentially the same result as obtained with the detailed integrations.

These robust dates could conceivably be moved around. One possibility is that there might be unknown physical mechanisms that have not been modeled. This is not so far-fetched, as recent studies of changing P in classical nova demonstrate that some unknown effects are making for substantial changes in the steady $\dot{P}$ and the change of P across nova eruptions as compared to the standard models. *If* such mechanisms are real, and if they scale with accretion rate, and if they are not covered by the robustness trait, and if such mechanisms start working in V Sge over the next few deacdes, then the 2083 date might have to be shifted somewhat. Another possibility is that the chaotic variability of V Sge might make for intermittent effects that will speed up (or slow down) the merger. The 2083 date is for the average accretion rate of V Sge. But if V Sge is in a high accretion state half the time and this makes for in-spiraling that is more than 2X as effective as during the low state, then the total in-spiral rate will be faster than the average leading to 2083. Such intermittent effects are hard to calculate. In all, the year 2083 is robust, but perhaps unknown complications will shift this year somewhat.

In all, the detailed physics model for V Sge yields a merge year of 2083, with an uncertainty of ±16 years.